SL Paper 1

Consider the following propositions.

p : *Students stay up late.*

q : *Students fall asleep in class.*

a. Write the following compound proposition in symbolic form. [2]

If students do not stay up late then they will not fall asleep in class.

b. Complete the following truth table. [3]

p	q	$\neg q$	$p \vee \neg q$	$\neg(p \vee \neg q)$
T	T			
T	F			
F	T			
F	F			

c. Write down a reason why the statement $\neg(p \vee \neg q)$ is not a contradiction. [1]

Consider the statement $p \Rightarrow q$.

If I break my arm, then it will hurt.

a. Write down in words, the inverse of $p \Rightarrow q$. [2]

b. Complete the following truth table. [2]

p	q	$p \Rightarrow q$	Inverse of $p \Rightarrow q$	Converse of $p \Rightarrow q$
T	T	T		
T	F	F		
F	T	T		
F	F	T		

c. State whether the converse and the inverse of an implication are logically equivalent. [2]

Justify your answer.

For events A and B, the probabilities are $\mathrm{P}(A) = \frac{4}{13}$ and $\mathrm{P}(B) = \frac{5}{13}$.

a. If events A and B are mutually exclusive, write down the value of $\mathrm{P}(A \cap B)$. [1]

b. If events A and B are independent, find the value of $\mathrm{P}(A \cap B)$. [2]

c. If $\mathrm{P}(A \cup B) = \frac{7}{13}$, find the value of $\mathrm{P}(A \cap B)$. [3]

The universal set U is the set of integers from 1 to 20 inclusive.

A and B are subsets of U where:

A is the set of even numbers between 7 and 17.

B is the set of multiples of 3.

a. List the elements of the following sets: [1]

A,

b. List the elements of the following sets: [1]

B,

c. List the elements of the following sets: [2]

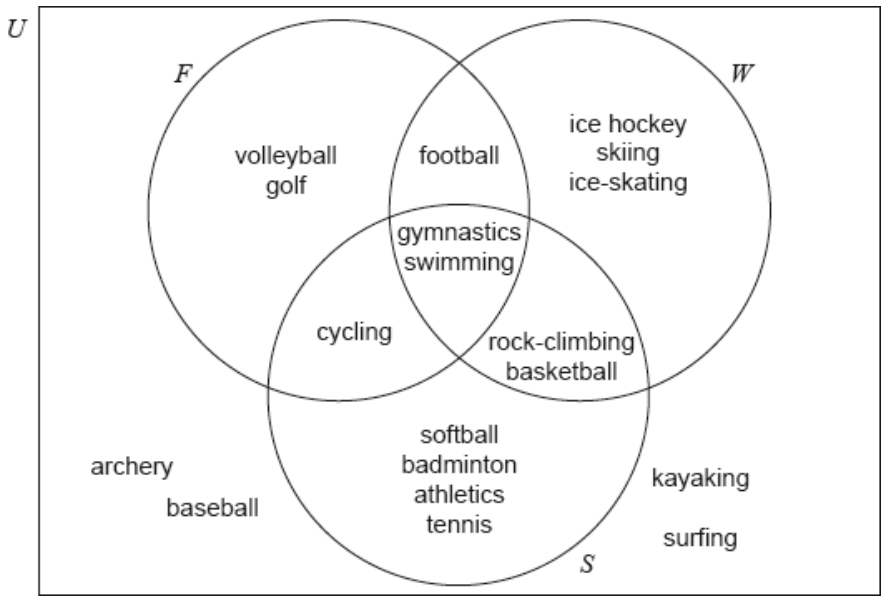
$A \cup B$,

d. List the elements of the following sets: [2]

$A \cap B'$.

Dune Canyon High School organizes its **school year** into three trimesters: fall/autumn (F), winter (W) and spring (S). The school offers a variety of sporting activities during and outside the school year.

The activities offered by the school are summarized in the following Venn diagram.



a. Write down the number of sporting activities offered by the school during its **school year**. [1]

b. Determine whether rock-climbing is offered by the school in the fall/autumn trimester. [1]

c.i. Write down the elements of the set $F \cap W'$; [1]

c.ii. Write down $n(W \cap S)$. [1]

d. Write down, in terms of F, W and S, an expression for the set which contains only archery, baseball, kayaking and surfing. [2]

Let p and q represent the propositions

p: food may be taken into the cinema

q: drinks may be taken into the cinema

a. Complete the truth table below for the symbolic statement $\neg(p \vee q)$. [2]

p	q	$p \vee q$	$\neg(p \vee q)$
T	T		
T	F		
F	T		
F	F		

b. Write down in words the meaning of the symbolic statement $\neg(p \vee q)$. [2]

c. Write in symbolic form the compound statement: [2]

"no food and no drinks may be taken into the cinema".

Consider the three propositions p, q and r.

p: *The food is well cooked*
q: *The drinks are chilled*
r: *Dinner is spoilt*

a. Write the following compound proposition in words. [3]

$$(p \wedge q) \Rightarrow \neg r$$

b. Complete the following truth table. [3]

p	q	r	$p \wedge q$	$\neg r$	$(p \wedge q) \Rightarrow \neg r$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

a. Consider the following propositions: [3]

p : The lesson is cancelled

q : The teacher is absent

r : The students are in the library.

Write, in words, the compound proposition $q \Rightarrow (p \wedge r)$.

b. Complete the following truth table. [2]

q	r	$\neg r$	$q \Rightarrow \neg r$
T	T		
T	F		
F	T		
F	F		

c. **Hence**, justify why $q \Rightarrow \neg r$ is not a tautology. [1]

You may choose from three courses on a lunchtime menu at a restaurant.

s: you choose a salad,

m: you choose a meat dish (main course),

d: you choose a dessert.

You choose a **two** course meal which **must** include a main course and either a salad or a dessert, but not both.

a. Write the sentence above using logic symbols. [2]

b. Write in words $s \Rightarrow \neg d$. [2]

c. Complete the following truth table. [2]

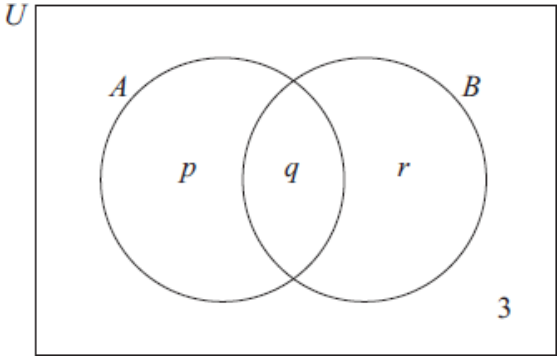
s	d	$\neg s$	$\neg s \Rightarrow d$
T	T		
T	F		
F	T		
F	F		

A group of 33 people was asked about the passports they have. 21 have Australian passports, 15 have British passports and 3 have neither.

A group of 33 people was asked about the passports they have. 21 have Australian passports, 15 have British passports and 3 have neither.

a. Find the number that have both Australian and British passports. [2]

b. In the Venn diagram below, set A represents the people in the group with Australian passports and set B those with British passports. [2]

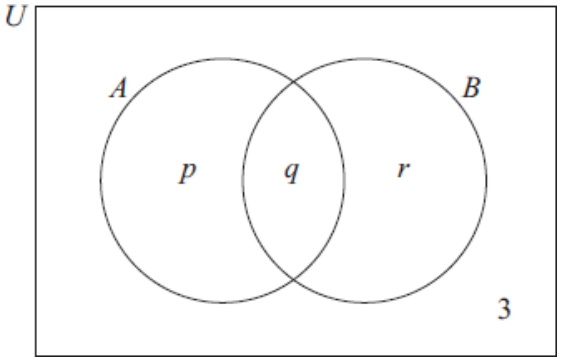


Write down the value of

(i) q ;

(ii) p and of r .

c. In the Venn diagram below, set A represents the people in the group with Australian passports and set B those with British passports. [2]



Find $n(A \cup B')$.

Police in a town are investigating the theft of mobile phones one evening from three cafés, "Alan's Diner", "Sarah's Snackbar" and "Pete's Eats".

They interviewed two suspects, Matthew and Anna, about that evening.

Matthew said:

"I visited Pete's Eats and visited Alan's Diner and I did not visit Sarah's Snackbar."

Let p , q and r be the statements:

p : I visited Alan's Diner
q : I visited Sarah's Snackbar
r : I visited Pete's Eats

a. Write down Matthew's statement in symbolic logic form. [3]

b. What Anna said was lost by the police, but in symbolic form it was [3]

$$(q \vee r) \Rightarrow \neg p$$

Write down, in words, what Anna said.

Consider the following propositions.

p : the baby cries
q : the baby is happy
r : the baby wants to play

a. Write down, in words, $(q \wedge r) \Rightarrow \neg p$. [3]

b. Complete the following truth table. [2]

p	q	r	$\neg p$	$(q \wedge r)$	$(q \wedge r) \Rightarrow \neg p$
T	T	T	F		
T	T	F	F		
T	F	T	F		
T	F	F	F		
F	T	T	T		
F	T	F	T		
F	F	T	T		
F	F	F	T		

c. State whether $(q \wedge r) \Rightarrow \neg p$ is a tautology, contradiction or neither. [1]

Consider the statements

p : The numbers x and y are both even.

q : The sum of x and y is an even number.

a. Write down, in words, the statement $p \Rightarrow q$. [2]

b. Write down, in words, the inverse of the statement $p \Rightarrow q$. [2]

c. State whether the inverse of the statement $p \Rightarrow q$ is always true. Justify your answer. [2]

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a. Complete the truth table. [2]

p	q	$\neg p$	$\neg p \vee q$
T	T		
T	F		
F	T		
F	F		

b. Consider the propositions p and q: [2]

p: x is a number less than 10.

q: x2 is a number greater than 100.

Write in words the compound proposition $\neg p \vee q$.

c. Using part (a), determine whether $\neg p \vee q$ is true or false, for the case where x is a number less than 10 and x^2 is a number greater than 100. [1]

d. Write down a value of x for which $\neg p \vee q$ is false. [1]

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p : x is a multiple of 12

q : x is a multiple of 6.

a. Write down in words $\neg p$. [1]

b. Write down in symbolic form the compound statement [2]

r : If x is a multiple of 12, then x is a multiple of 6.

c. Consider the compound statement [1]

s : If x is a multiple of 6, then x is a multiple of 12.

Identify whether s : is the inverse, the converse or the contrapositive of r.

d. Consider the compound statement [2]

s : If x is a multiple of 6, then x is a multiple of 12.

Determine the validity of s. Justify your decision.

Consider the statement p:

"If a quadrilateral is a square then the four sides of the quadrilateral are equal".

a. Write down the inverse of statement *p* in words. [2]

b. Write down the converse of statement *p* in words. [2]

c. Determine whether the converse of statement *p* is always true. Give an example to justify your answer. [2]

The truth table below shows the truth-values for the proposition

$p \veebar q \Rightarrow \neg p \veebar \neg q$

p	q	$\neg p$	$\neg q$	$p \veebar q$	$\neg p \veebar \neg q$	$p \veebar q \Rightarrow \neg p \veebar \neg q$
T	T	F	F		F	
T	F	F		T	T	T
F	T	T	F	T	T	T
F	F	T	T	F		T

a. Explain the distinction between the compound propositions, $p \veebar q$ and $p \vee q$. [1]

b. Fill in the four missing truth-values on the table. [4]

c. State whether the proposition $p \veebar q \Rightarrow \neg p \veebar \neg q$ is a tautology, a contradiction or neither. [1]

Consider the propositions p and q.

p: *I take swimming lessons*

q: *I can swim 50 metres*

a. Complete the truth table below. [2]

p	q	$\neg q$	$p \vee \neg q$
T	T		
T	F		
F	T		
F	F		

b. Write the following compound proposition in symbolic form. [2]

"I cannot swim 50 metres and I take swimming lessons."

c. Write the following compound proposition in words. [2]

$q \Rightarrow \neg q$

Consider the universal set $U = \{x \in \mathbb{N} | 3 < x < 13\}$, and the subsets $A = \{\text{multiples of } 3\}$ and $B = \{4,\ 6,\ 12\}$.

a.i. List the elements of the following set. [1]

A

a.ii. List the elements of the following set. [1]

$A \cap B'$

b. Write down one element of $(A \cup B)'$. [2]

c. One of the statements in the table below is false. Indicate with an **X** which statement is false. Give a reason for your answer. [2]

$n(A \cup B) = 4$	
$15 \in A'$	
$A \subset A \cup B$	

Consider the two propositions p and q.

p: The sun is shining q: I will go swimming

a. Write in words the compound proposition [2]

$p \Rightarrow q$;

b. Write in words the compound proposition [2]

$\neg p \vee q$.

c. The truth table for these compound propositions is given below. [1]

p	q	$p \Rightarrow q$	$\neg p$	$\neg p \vee q$
T	T	T		T
T	F	F		F
F	T	T		T
F	F	T		T

Complete the column for $\neg p$.

d. The truth table for these compound propositions is given below. [1]

p	q	$p \Rightarrow q$	$\neg p$	$\neg p \vee q$
T	T	T		T
T	F	F		F
F	T	T		T
F	F	T		T

State the relationship between the compound propositions $p \Rightarrow q$ and $\neg p \vee q$.

Two propositions p and q are defined as follows

p: Eva is on a diet

q: Eva is losing weight.

a. Write down the following statement **in words**. [2]

$$q \Rightarrow p$$

b. Write down, in words, the contrapositive statement of $q \Rightarrow p$. [2]

c. Determine whether your statement in part (a) is logically equivalent to your statement in part (b). Justify your answer. [2]

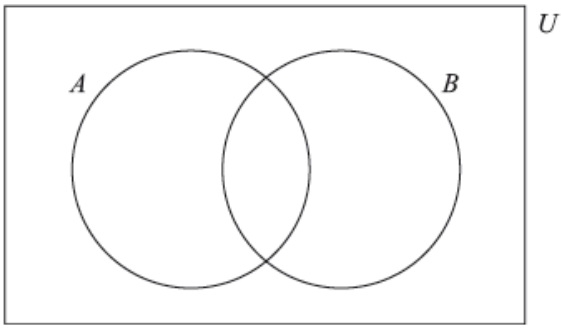
Aleph has an unbiased cubical (six faced) die on which are written the numbers

1 , 2 , 3 , 4 , 5 and 6.

Beth has an unbiased tetrahedral (four faced) die on which are written the numbers

2 , 3 , 5 and 7.

a. Complete the Venn diagram with the numbers written on Aleph's die (A) and Beth's die (B). [2]



b. Find $n(B \cap A')$. [2]

c. Aleph and Beth are each going to roll their die once only. Shin says the probability that each die will show the same number is $\frac{1}{8}$. [2]

Determine whether Shin is correct. Give a reason.

Consider the following Venn diagrams. Each diagram is shaded differently.

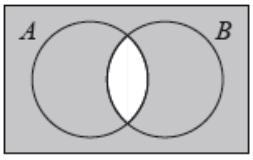


Diagram 1

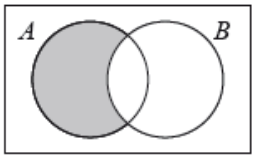


Diagram 2

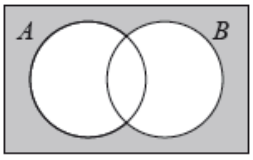


Diagram 3

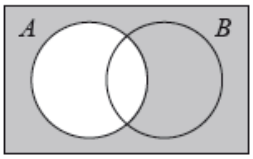


Diagram 4

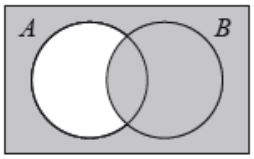


Diagram 5

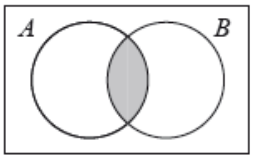


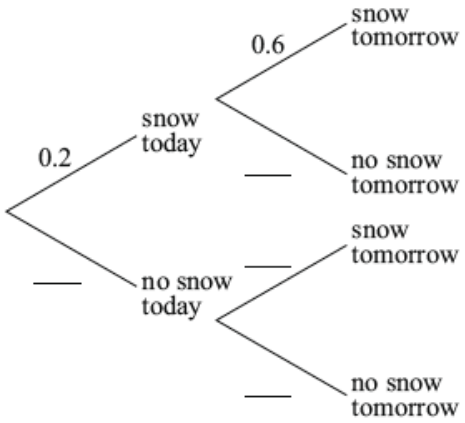
Diagram 6

In the following table there are six sets. Each of these sets corresponds to the shaded region of one of the Venn diagrams. In the correct space, write the number of the diagram that corresponds to that set.

Set	Diagram
$(A \cup B)'$	. . . . .
$A' \cup B'$	. . . . .
$A \cap B'$	. . . . .
$A \cap B$	. . . . .
$A' \cup B$	. . . . .
A'	. . . . .

The probability that it snows today is 0.2. If it does snow today, the probability that it will snow tomorrow is 0.6. If it does not snow today, the probability that it will not snow tomorrow is 0.9.

a. Using the information given, complete the following tree diagram. [3]



b. Calculate the probability that it will snow tomorrow. [3]

Consider two propositions p and q.

a. Complete the truth table below. [4]

p	q	$\neg q$	$p \Rightarrow \neg q$	$\neg p$	$\neg p \Rightarrow q$
T	T				
T	F				
F	T				
F	F				

b. Decide whether the compound proposition [2]

$$(p \Rightarrow \neg q) \Leftrightarrow (\neg p \Rightarrow q)$$

is a tautology. State the reason for your decision.

Alan's laundry basket contains two green, three red and seven black socks. He selects one sock from the laundry basket at random.

a. Write down the probability that the sock is red. [1]

b. Alan returns the sock to the laundry basket and selects two socks at random. [2]

Find the probability that the first sock he selects is green and the second sock is black.

c. Alan returns the socks to the laundry basket and again selects two socks at random. [3]

Find the probability that he selects two socks of the same colour.

U is the set of **positive** integers less than or equal to 10.

A, B and C are subsets of U.

$A = \{\text{even integers}\}$

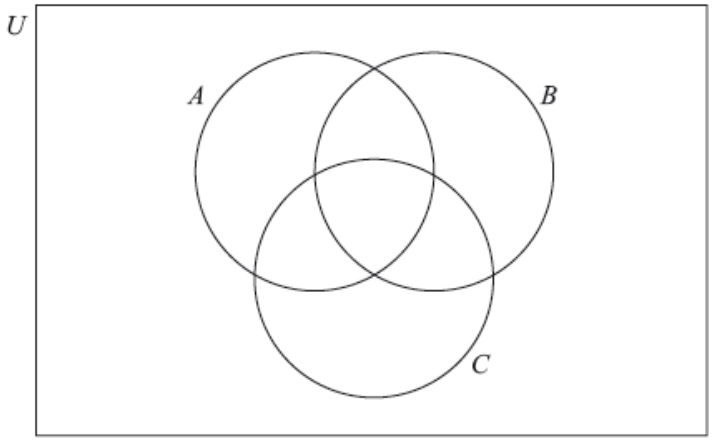
$B = \{\text{multiples of } 3\}$

$C = \{6,\ 7,\ 8,\ 9\}$

a. List the elements of A. [1]

b. List the elements of B. [1]

c. Complete the Venn diagram with **all** the elements of U. [4]



A survey was carried out at an international airport. A number of travellers were interviewed and asked for their flight destinations. The results are shown in the table below.

Destination	America	Africa	Asia
Number of males	45	62	37
Number of females	35	46	25

a. One traveller is to be chosen at random from all those interviewed. [2]

Find the probability that this traveller was going to Africa.

b. One female traveller is to be chosen at random from all those interviewed. [2]

Find the probability that this female traveller was going to Asia.

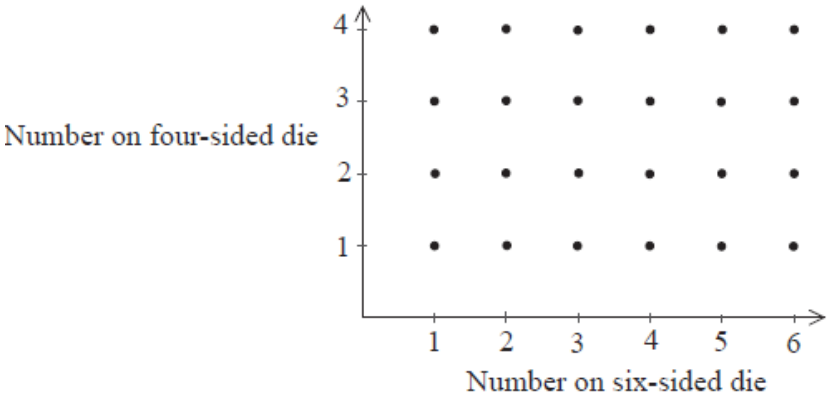
c. One traveller is to be chosen at random from those **not** going to America. [2]

Find the probability that the chosen traveller is female.

A fair six-sided die has the numbers 1, 2, 3, 4, 5, 6 written on its faces. A fair four-sided die has the numbers 1, 2, 3, and 4 written on its faces.

The two dice are rolled.

The following diagram shows the possible outcomes.



a. Find the probability that the two dice show the same number. [2]

b. Find the probability that the difference between the two numbers shown on the dice is 1. [2]

c. Find the probability that the number shown on the four-sided die is greater than the number shown on the six-sided die, given that the difference between the two numbers is 1. [2]

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a. Complete the following truth table. [2]

p	q	.....	$p \Rightarrow \neg q$
T	T	F	.....
T	F	T	.....
F	T	F	.....
F	F	T	.....

b. Consider the propositions [2]

p: *Cristina understands logic*

q: *Cristina will do well on the logic test.*

Write down the following compound proposition in symbolic form.

"If Cristina understands logic then she will do well on the logic test"

c. Write down in words the contrapositive of the proposition given in part (b). [2]

In a particular school, students must choose at least one of three optional subjects: art, psychology or history.

Consider the following propositions

a: I choose art,
p: I choose psychology,
h: I choose history.

a. Write, in words, the compound proposition [3]

$$\neg h \Rightarrow (p \vee a)$$

.

b. Complete the truth table for $\neg a \Rightarrow p$. [1]

a	p	$\neg a$	$\neg a \Rightarrow p$
T	T	F	
T	F	F	
F	T	T	
F	F	T	

c. State whether $\neg a \Rightarrow p$ is a tautology, a contradiction or neither. Justify your answer. [2]

Two propositions are defined as follows:

p : *Quadrilateral ABCD has two diagonals that are equal in length.*

q : *Quadrilateral ABCD is a rectangle.*

a. Express the following in symbolic form. [2]

"A rectangle always has two diagonals that are equal in length."

b. Write down in symbolic form the converse of the statement in (a). [1]

c. Determine, **without** using a truth table, whether the statements in (a) and (b) are logically equivalent. [2]

d. Write down the name of the statement that is logically equivalent to the converse. [1]

Consider the following propositions.

p: The car is under warranty
q: The car is less than 2 years old
r : The car has been driven more than 20 000 km

a. Write down in words $(q \vee \neg r) \Rightarrow p$. [3]

b. Complete the truth table. [2]

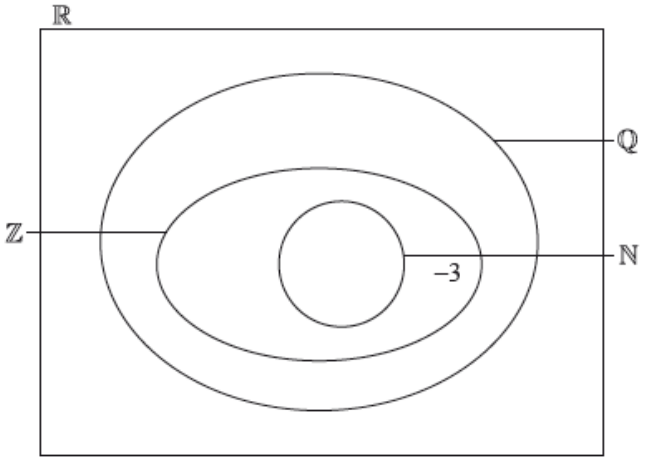
p	q	r	$\neg r$	$q \vee \neg r$	$(q \vee \neg r) \Rightarrow p$
T	T	T	F		
T	T	F	T		
T	F	T	F		
T	F	F	T		
F	T	T	F		
F	T	F	T		
F	F	T	F		
F	F	F	T		

c. State whether the statement $\neg p \Rightarrow \neg(q \vee \neg r)$ is the inverse, the converse or the contrapositive of the statement in part (a). [1]

The following Venn diagram shows the relationship between the sets of numbers

$$\mathbb{N},\ \mathbb{Z},\ \mathbb{Q} \text{ and } \mathbb{R}.$$

The number –3 belongs to the set of $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$, but not $\mathbb{N}$, and is placed in the appropriate position on the Venn diagram as an example.



Write down the following numbers in the appropriate place in the Venn diagram.

a. 4 [1]

b. $\frac{1}{3}$ [1]

c. π [1]

d. 0.38 [1]

e. $\sqrt{5}$ [1]

f. -0.25 [1]

Consider the following logic propositions:

p : Sean is at school

q : Sean is playing a game on his computer.

a. Write in words, $p \veebar q$. [2]

b. Write in words, the converse of $p \Rightarrow \neg q$. [2]

c. Complete the following truth table for $p \Rightarrow \neg q$. [2]

p	q	$\neg q$	$p \Rightarrow \neg q$
T	T		
T	F		
F	T		
F	F		

Consider the following logic propositions:

p : *Yuiko is studying French.*

q : *Yuiko is studying Chinese.*

a. Write down the following compound propositions in symbolic form. [3]

(i) Yuiko is studying French but not Chinese.

(ii) Yuiko is studying French or Chinese, but not both.

b. Write down in words the **inverse** of the following compound proposition. [3]

If Yuiko is studying Chinese, then she is not studying French.

A class consists of students studying Spanish or French or both. Fifteen students study Spanish and twelve study French.

The probability that a student studies French given that she studies Spanish is $\frac{7}{15}$.

a. Draw a Venn diagram in the space below to illustrate this information. [3]

b. Find the probability that a student studies Spanish given that she studies one language only. [3]

Consider the following logic propositions.

p: Sandi gets up before eight o'clock

q: Sandi goes for a run

r: Sandi goes for a swim

a. Write down in words the compound proposition [3]

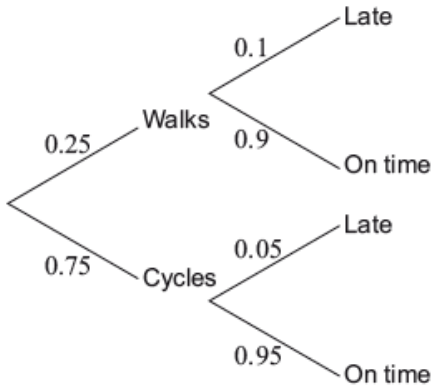
$p \Rightarrow (q \veebar r)$.

b. Complete the following truth table. [2]

p	q	r	$q \veebar r$	$p \Rightarrow (q \veebar r)$
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

c. On a morning when Sandi does **not** get up before eight o'clock, use your truth table to determine whether $p \Rightarrow (q \veebar r)$ is a tautology, contradiction or neither. [1]

Peter either walks or cycles to work. The probability that he walks is 0.25. If Peter walks to work, the probability that he is late is 0.1. If he cycles to work, the probability that he is late is 0.05. The tree diagram for this information is shown.



a. On a day chosen at random, Peter walked to work. [1]

Write down the probability that he was on time.

b. For a different day, also chosen at random, [2]

find the probability that Peter cycled to work and was late.

c. For a different day, also chosen at random, [3]

find the probability that, given Peter was late, he cycled to work.

A bag contains 7 red discs and 4 blue discs. Ju Shen chooses a disc at random from the bag and removes it. Ramón then chooses a disc from those left in the bag.

a. Write down the probability that [3]

(i) Ju Shen chooses a red disc from the bag;

(ii) Ramón chooses a blue disc from the bag, given that Ju Shen has chosen a red disc;

(iii) Ju Shen chooses a red disc and Ramón chooses a blue disc from the bag.

b. Find the probability that Ju Shen and Ramón choose different coloured discs from the bag. [3]

The Home Shine factory produces light bulbs, 7% of which are found to be defective.

Francesco buys two light bulbs produced by Home Shine.

The Bright Light factory also produces light bulbs. The probability that a light bulb produced by Bright Light is not defective is a.

Deborah buys three light bulbs produced by Bright Light.

a. Write down the probability that a light bulb produced by Home Shine is not defective. [1]

b.i. Find the probability that both light bulbs are not defective. [2]

b.ii. Find the probability that at least one of Francesco's light bulbs is defective. [2]

c. Write down an expression, in terms of a, for the probability that at least one of Deborah's three light bulbs is defective. [1]

In a research project on the relation between the gender of 150 science students at college and their degree subject, the following set of data is collected.

		Degree Subject		
		Biology	Physics	Chemistry
Gender	Male	40	16	35
	Female	15	24	20

a. Find the probability that a student chosen at random is male. [2]

b. Find the probability that a student chosen at random is either male or studies Chemistry. [2]

c. Find the probability that a student chosen at random studies Physics, given that the student is male. [2]

---

a. Complete the truth table shown below. [3]

p	q	$p \wedge q$	$p \vee (p \wedge q)$	$(p \vee (p \wedge q)) \Rightarrow p$
T	T			
T	F			
F	T			
F	F			

b. State whether the compound proposition $(p \vee (p \wedge q)) \Rightarrow p$ is a contradiction, a tautology or neither. [1]

c. Consider the following propositions. [2]

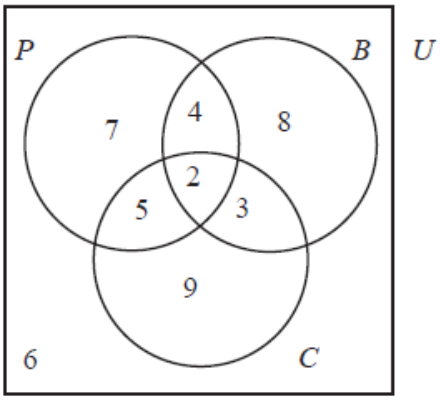
p: Feng finishes his homework

q: Feng goes to the football match

Write in symbolic form the following proposition.

If Feng does not go to the football match then Feng finishes his homework.

The Venn diagram shows the numbers of pupils in a school according to whether they study the sciences Physics (P), Chemistry (C), Biology (B).



a. Write down the number of pupils that study Chemistry only. [1]

b. Write down the number of pupils that study **exactly** two sciences. [1]

c. Write down the number of pupils that do not study Physics. [2]

d. Find $n[(P \cup B) \cap C]$. [2]

Consider the following logic statements.

p: Carlos is playing the guitar

q: Carlos is studying for his IB exams

a. Write in words the compound statement $\neg p \wedge q$. [2]

b. Write the following statement in symbolic form. [1]

"Either Carlos is playing the guitar or he is studying for his IB exams but not both."

c. Write the **converse** of the following statement in **symbolic form**. [3]

"If Carlos is playing the guitar then he is not studying for his IB exams."

In the Canadian city of Ottawa:

$$97\% \text{ of the population speak English,}$$
$$38\% \text{ of the population speak French,}$$
$$36\% \text{ of the population speak both English and French.}$$

The total population of Ottawa is $985\,000$.

a. Calculate the percentage of the population of Ottawa that speak English but not French. [2]

b. Calculate the number of people in Ottawa that speak both English and French. [2]

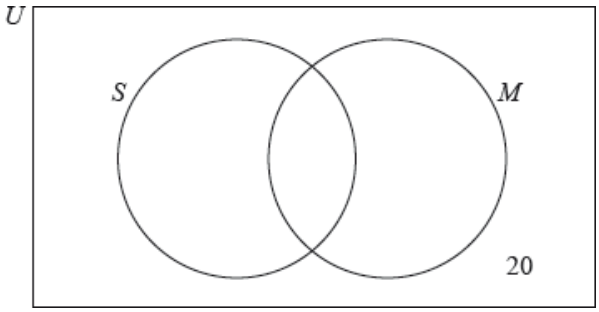
c. Write down your answer to part (b) in the form $a \times 10^k$ where $1 \leqslant a < 10$ and $k \in \mathbb{Z}$. [2]

Rosewood College has 120 students. The students can join the sports club (S) and the music club (M).

For a student chosen at random from these 120, the probability that they joined both clubs is $\frac{1}{4}$ and the probability that they joined the music club is $\frac{1}{3}$.

There are 20 students that did not join either club.

a. Complete the Venn diagram for these students. [2]



b. One of the students who joined the sports club is chosen at random. Find the probability that this student joined both clubs. [2]

c. Determine whether the events S and M are independent. [2]

Consider the propositions

p: *I have a bowl of soup.*
q: *I have an ice cream.*

a. Write down, in words, the compound proposition $\neg p \Rightarrow q$. [2]

b. Complete the truth table. [2]

p	q	$\neg p$	$\neg p \Rightarrow q$
T	T		
T	F		
F	T		
F	F		

c. Write down, in symbolic form, the converse of $\neg p \Rightarrow q$. [2]

Consider the following propositions.

p: I completed the task
q: I was paid

a. Write down in words $\neg q$. [1]

b. Write down in symbolic form the compound statement: [1]

If I was paid then I completed the task.

c.i. Complete the following truth table. [2]

p	q	$\neg q$	$p \vee \neg q$	$q \Rightarrow p$
T	T	F		
T	F	T		
F	T	F		
F	F	T		

c.ii. State whether the statements $p \vee \neg q$ and $q \Rightarrow p$ are logically equivalent. Give a reason for your answer. [2]

Let $\mathrm{P}(A) = 0.5$, $\mathrm{P}(B) = 0.6$ and $\mathrm{P}(A \cup B) = 0.8$.

a. Find $\mathrm{P}(A \cap B)$. [2]

b. Find $\mathrm{P}(A|B)$. [2]

c. Decide whether *A* and *B* are independent events. Give a reason for your answer. [2]

---

a. Two friends, Sensen and Cruz, are conducting an investigation on probability. [2]

Sensen has a fair six-sided die with faces numbered $1,\ 2,\ 2,\ 4,\ 4$ and 4. Cruz has a fair disc with one red side and one blue side.

The die and the disc are thrown at the same time.

Find the probability that the number shown on the die is 1 **and** the colour shown on the disc is blue;

b. Find the probability that the number shown on the die is 1 **or** the colour shown on the disc is blue; [2]

c. Find the probability that the number shown on the die is even given that the colour shown on the disc is red. [2]

Consider the propositions r, p and q.

a. Complete the following truth table. [4]

r	p	q	$r \wedge p$	$\neg q$	$(r \wedge p) \vee \neg q$	$\neg\big((r \wedge p) \vee \neg q\big)$	$\neg(r \wedge p)$	$\neg(r \wedge p) \wedge q$
T	T	T		F			F	
T	T	F		T			F	
T	F	T		F			T	
T	F	F		T			T	
F	T	T		F			T	
F	T	F		T			T	
F	F	T		F			T	
F	F	F		T			T	

b. Determine whether the compound proposition $\neg\left((r \wedge p) \vee \neg q\right) \Leftrightarrow \neg(r \wedge p) \wedge q$ is a tautology, a contradiction or neither. [2]

Give a reason.

---

a. Consider the following statements [2]

z : x is an integer
q : x is a rational number
r : x is a real number.

i) Write down, in words, $\neg q$.

ii) Write down a value for x such that the statement $\neg q$ is true.

b. Write the following argument in symbolic form: [3]

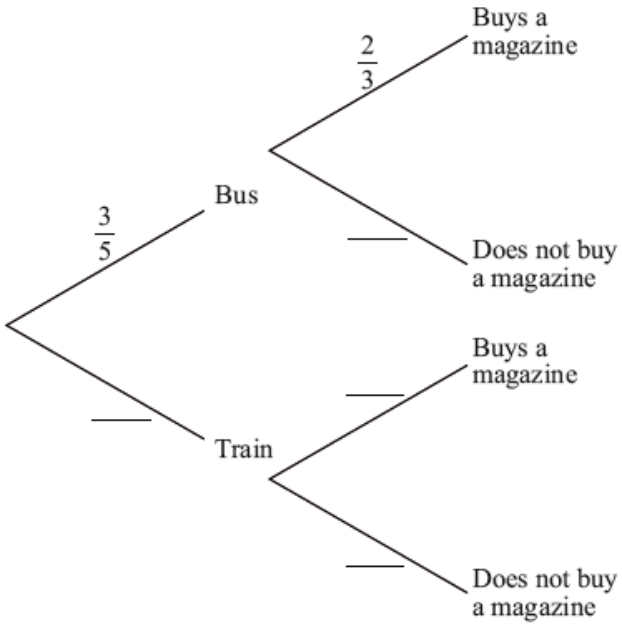
"If x is a real number and x is not a rational number, then x is not an integer".

c. Phoebe states that the argument in part (b) can be shown to be valid, without the need of a truth table. [1]

Justify Phoebe's statement.

Ramzi travels to work each day, either by bus or by train. The probability that he travels by bus is $\frac{3}{5}$. If he travels by bus, the probability that he buys a magazine is $\frac{2}{3}$. If he travels by train, the probability that he buys a magazine is $\frac{3}{4}$.

a. Complete the tree diagram. [3]



b. Find the probability that Ramzi buys a magazine when he travels to work. [3]

B and C are subsets of a universal set U such that

$$U = \{x : x \in \mathbb{Z}, 0 \leqslant x < 10\},\ B = \{\text{prime numbers} < 10\},\ C = \{x : x \in \mathbb{Z}, 1 < x \leqslant 6\}.$$

a. List the members of sets [4]

(i) B

(ii) $C \cap B$

(iii) $B \cup C'$

b. Consider the propositions: [2]

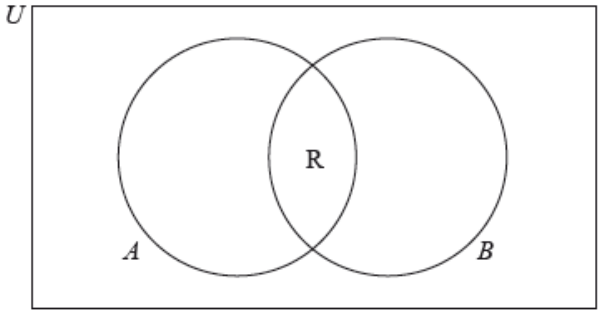
p : *x* is a prime number less than 10.

q : *x* is a positive integer between 1 and 7.

Write down, in words, the contrapositive of the statement, “If *x* is a prime number less than 10, then *x* is a positive integer between 1 and 7.”

Tuti has the following polygons to classify: rectangle (R), rhombus (H), isosceles triangle (I), regular pentagon (P), and scalene triangle (T).

In the Venn diagram below, set A consists of the polygons that have at least one pair of parallel sides, and set B consists of the polygons that have at least one pair of equal sides.



a. Complete the Venn diagram by placing the letter corresponding to each polygon in the appropriate region. For example, R has already been placed, and represents the rectangle. [3]

b. State which polygons from Tuti's list are elements of [3]

(i) $A \cap B$;

(ii) $(A \cup B)'$.

Consider the following statements about the quadrilateral ABCD

q : ABCD has four equal sides s : ABCD is a square

a. Express in words the statement, $s \Rightarrow q$. [2]

b. Write down in words, the inverse of the statement, $s \Rightarrow q$. [2]

c. Determine the validity of the argument in (b). Give a reason for your decision. [2]

A survey was carried out in a group of 200 people. They were asked whether they smoke or not. The collected information was organized in the following table.

	Smoker	Non-smoker
Male	60	40
Female	30	70

One person from this group is chosen at random.

a. Write down the probability that this person is a smoker. [2]

b. Write down the probability that this person is male given that they are a smoker. [2]

c. Find the probability that this person is a smoker or is male. [2]

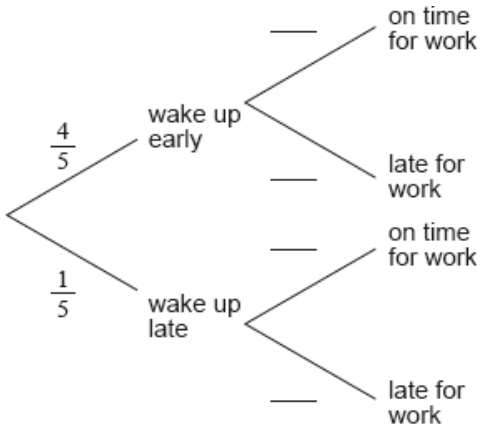
On a work day, the probability that Mr Van Winkel wakes up early is $\frac{4}{5}$.

If he wakes up early, the probability that he is on time for work is p.

If he wakes up late, the probability that he is on time for work is $\frac{1}{4}$.

The probability that Mr Van Winkel arrives on time for work is $\frac{3}{5}$.

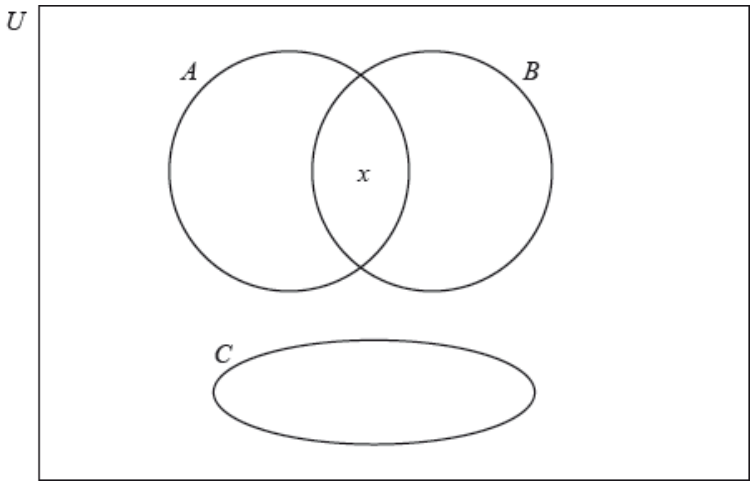
a. Complete the tree diagram below. [2]



b. Find the value of p. [4]

The following Venn diagram shows the sets A, B, C and U.

x is an element of U.



a. In the table indicate whether the given statements are True or False. [5]

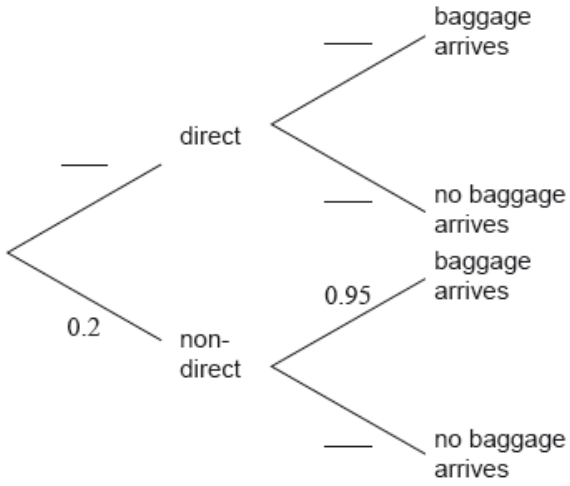
Statement	True or False
$x \in C$	
$x \subset B$	
$A \cup B \neq \varnothing$	
$A \cap B \subset C$	
$A \cap C = \varnothing$	

b. On the Venn diagram, shade the region $A \cap (B \cup C)'$. [1]

Sara regularly flies from Geneva to London. She takes either a direct flight or a non-directflight that goes via Amsterdam.

If she takes a direct flight, the probability that her baggage does not arrive in London is 0.01.
If she takes a non-direct flight the probability that her baggage arrives in London is 0.95.

The probability that she takes a non-direct flight is 0.2.



a. Complete the tree diagram. [3]

b. Find the probability that Sara's baggage arrives in London. [3]

The IB grades attained by a group of students are listed as follows.

$$6 \quad 4 \quad 5 \quad 3 \quad 7 \quad 3 \quad 5 \quad 4 \quad 2 \quad 5$$

a. Find the median grade. [2]

b. Calculate the interquartile range. [2]

c. Find the probability that a student chosen at random from the group scored at least a grade 4. [2]

A **weighted** die has 2 red faces, 3 green faces and 1 black face. When the die is thrown, the black face is three times as likely to appear on top as one of the other five faces. The other five faces have equal probability of appearing on top.

The following table gives the probabilities.

Red 1	Red 2	Green 1	Green 2	Green 3	Black
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{m}{8}$	$\frac{1}{8}$	$\frac{n}{8}$

a. Find the value of [2]

(i) m;

(ii) n.

b. The die is thrown once. [2]

Given that the face on top is not red, find the probability that it is black.

c. The die is now thrown twice. [2]

Calculate the probability that black appears on top both times.

---

a. (i) Complete the truth table below. [4]

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T			F	F	
T	F			F	T	
F	T			T	F	
F	F			T	T	

(ii) State whether the compound propositions $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are equivalent.

b. Consider the following propositions. [2]

p : Amy eats sweets

q : Amy goes swimming.

Write, in symbolic form, the following proposition.

Amy either eats sweets or goes swimming, but not both.

Consider each of the following statements

$$p : Alex\ is\ from\ Uruguay$$

$$q : Alex\ is\ a\ scientist$$

$$r : Alex\ plays\ the\ flute$$

a. Write the following argument in words [3]

$$\neg r \Rightarrow (q \vee p)$$

b. Complete the truth table for the argument in part (a) using the values below for p , q , r and $\neg r$. [2]

p	q	r	$\neg r$	$q \vee p$	$\neg r \Rightarrow (q \vee p)$
T	T	T	F		
T	T	F	T		
T	F	T	F		
T	F	F	T		
F	T	T	F		
F	T	F	T		
F	F	T	F		
F	F	F	T		

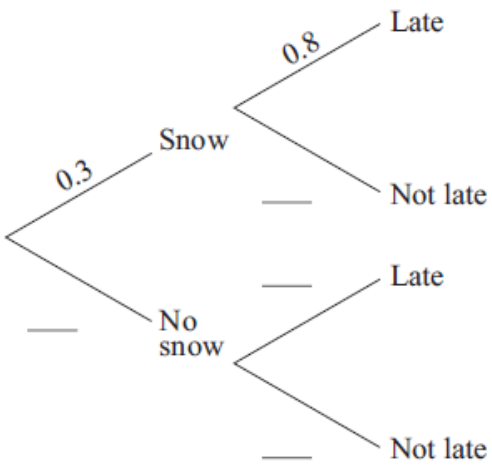
c. The argument $\neg r \Rightarrow (q \vee p)$ is invalid. State the reason for this. [1]

The probability that it will snow tomorrow is 0.3.

If it snows tomorrow the probability that Chuck will be late for school is 0.8.

If it does not snow tomorrow the probability that Chuck will be late for school is 0.1.

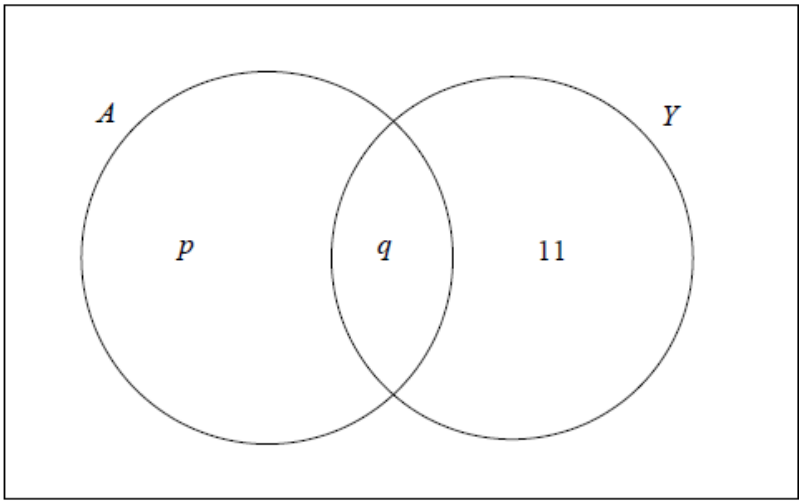
a. Complete the tree diagram below. [3]



b. Find the probability that it does not snow tomorrow and Chuck is late for school. [1]

c. Find the probability that Chuck is late for school. [2]

A fitness club has 60 members. 35 of the members attend the club's aerobics course (A) and 28 members attend the club's yoga course (Y). 17 members attend both courses. A Venn diagram is used to illustrate this situation.



a. Write down the value of q. [1]

b. Find the value of p. [2]

c. Calculate the number of members of the fitness club who attend neither the aerobics course (A) nor the yoga course (Y). [2]

d. Shade, on your Venn diagram, $A' \cap Y$. [1]

---

a. Complete the truth table below. [4]

p	q	$\neg p$	$(p \wedge q)$	$(\neg p \veebar q)$	$(p \wedge q) \Rightarrow (\neg p \veebar q)$
T	T				
T	F				
F	T				
F	F				

b.i. State whether the statement $(p \wedge q) \Rightarrow (\neg p \veebar q)$ is a logical contradiction, a tautology or neither. [1]

b.ii. Give a reason for your answer to part (b)(i). [1]

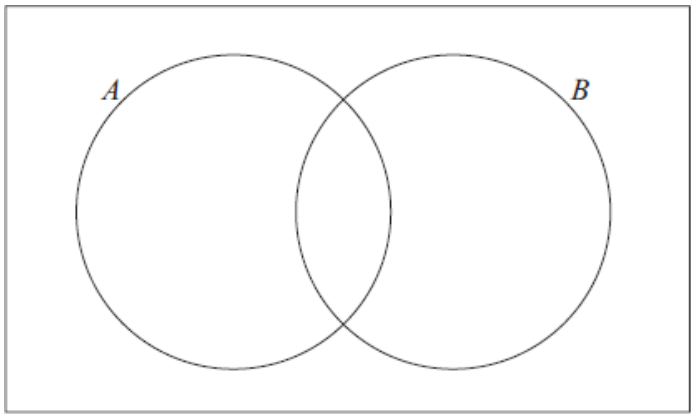
A group of 30 students were asked about their favourite topping for toast.

18 liked peanut butter (A)

10 liked jam (B)

6 liked neither

a. Show this information on the Venn diagram below. [2]



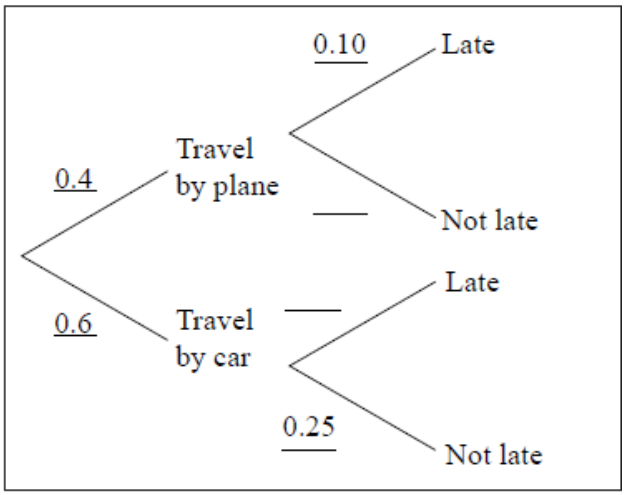
b. Find the number of students who like both peanut butter and jam. [2]

c. Find the probability that a randomly chosen student from the group likes peanut butter, given that they like jam. [2]

Merryn plans to travel to a concert tomorrow. Due to bad weather, there is a 60 % chance that all flights will be cancelled tomorrow. If the flights are cancelled Merryn will travel by car.

If she travels by plane the probability that she **will be late** for the concert is 10 %.
If she travels by car, the probability that she **will not be late** for the concert is 25 %.

a. Complete the tree diagram below. [1]



b. Find the probability that Merryn will not be late for the concert. [3]

c. Merryn was not late for the concert the next day. [2]

Given that, find the probability that she travelled to the concert by car.

$U = \{x | x \text{ is an integer}, 2 < x < 10\}$

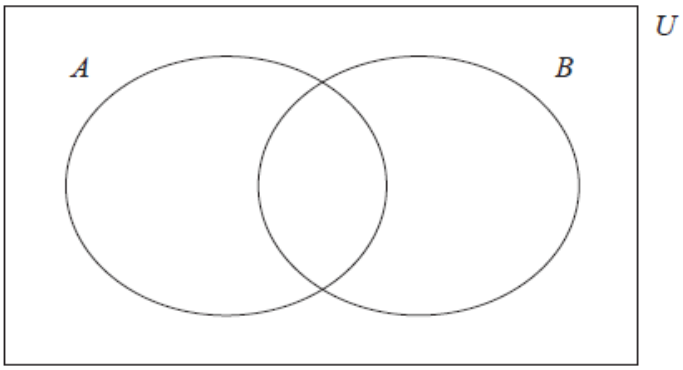
A and B are subsets of U such that $A = \{\text{multiples of } 3\}$, $B = \{\text{factors of } 24\}$.

a. List the elements of [2]

(i) U ;

(ii) B .

b. Write down the elements of U on the Venn diagram. [3]



c. Write down $n(A \cap B)$. [1]

When Andy plays tennis, 65% of his first serves go into the correct area of the court.

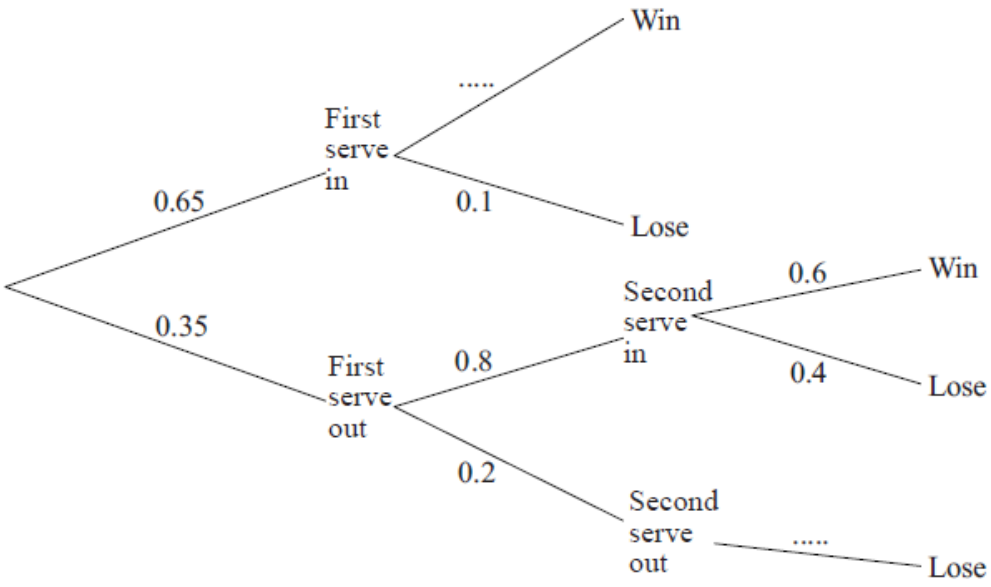
If the first serve goes into the correct area, his chance of winning the point is 90%.

If his first serve does not go into the correct area, Andy is allowed a second serve and, of these, 80% go into the correct area.

If the second serve goes into the correct area, his chance of winning the point is 60%.

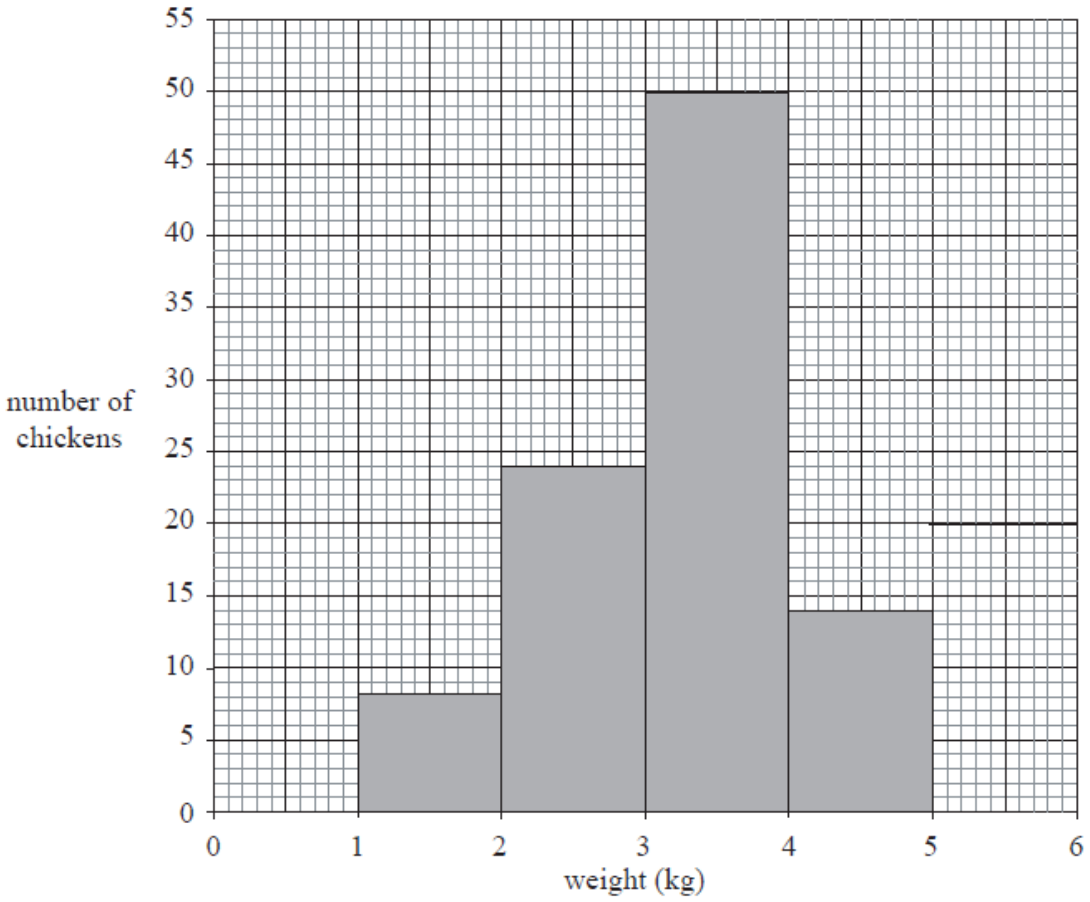
If neither serve goes into the correct area, Andy loses the point.

a. Complete the tree diagram below. [2]



b. Find the probability that Andy loses the point. [4]

The following histogram shows the weights of a number of frozen chickens in a supermarket. The weights are grouped such that $1 \leqslant \text{weight} < 2$, $2 \leqslant \text{weight} < 3$ and so on.



b. Find the total number of chickens. [1]

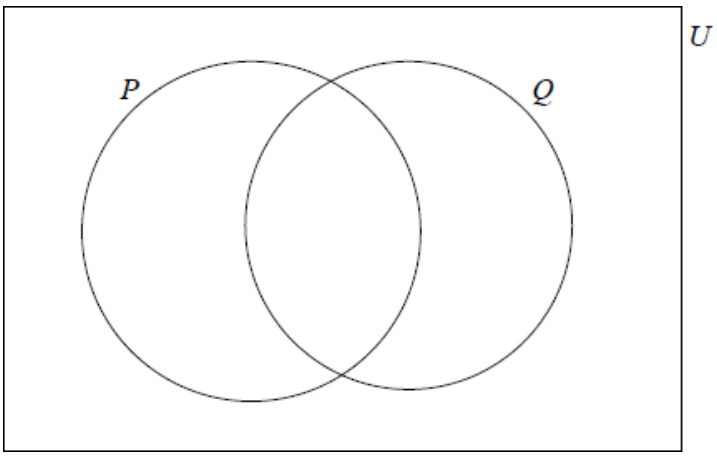
c. Write down the modal group. [1]

d. Gabriel chooses a chicken at random. [2]

Find the probability that this chicken weighs less than 4 kg.

The sets P, Q and U are defined as

U = {Real Numbers} , P = {Positive Numbers} and Q = {Rational Numbers}.

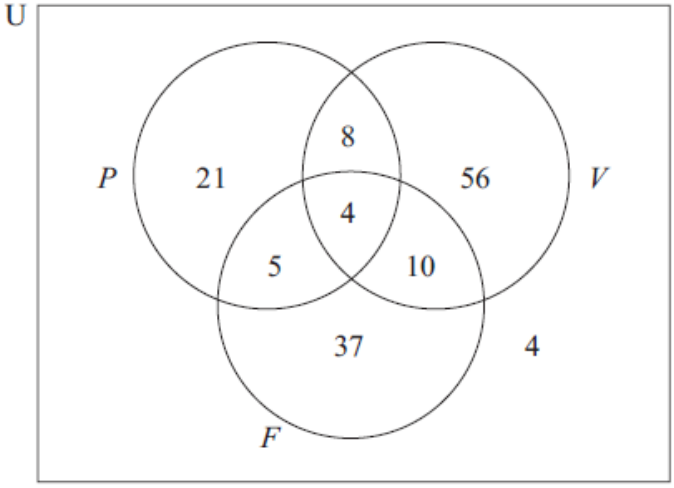


Write down in the correct region on the Venn diagram the numbers

$\frac{22}{7}$, 5×10^{-2} , $\sin(60^\circ)$, 0 , $\sqrt[3]{-8}$, $-\pi$.

Music lessons in Piano (P), Violin (V) and Flute (F) are offered to students at a school.

The Venn diagram shows the number of students who learn each kind of instrument.



a. Write down the total number of students in the school. [1]

b. Write down the number of students who [3]

(i) learn violin only;

(ii) learn piano or flute or both;

(iii) do not learn flute.

c. Explain, in words, the meaning of the part of the diagram that represents the set $P \cap F'$. [2]

In an international competition, participants can answer questions in **only one** of the three following languages: Portuguese, Mandarin or Hindi. 80 participants took part in the competition. The number of participants answering in Portuguese, Mandarin or Hindi is shown in the table.

		Languages			
		Portuguese	Mandarin	Hindi	**Total**
Participants	Boys	20	18	5	**43**
	Girls	18	7	12	**37**
	Total	**38**	**25**	**17**	**80**

A boy is chosen at random.

a. State the number of boys who answered questions in Portuguese. [1]

b. Find the probability that the boy answered questions in Hindi. [2]

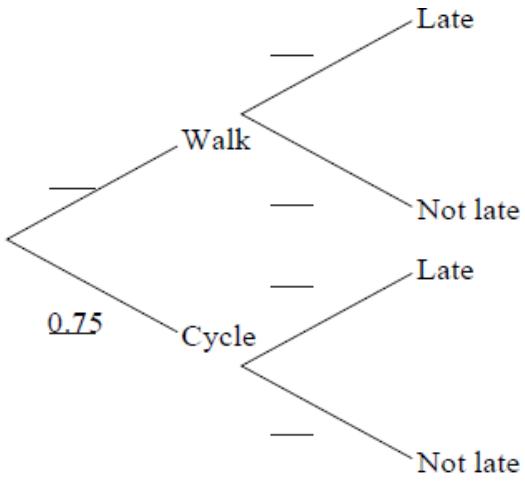
c. Two girls are selected at random. [3]

Calculate the probability that one girl answered questions in Mandarin and the other answered questions in Hindi.

Maria travels to school either by walking or by bicycle. The probability she cycles to school is 0.75.

If she walks, the probability that she is late for school is 0.1.
If she cycles, the probability that she is late for school is 0.05.

a. Complete the tree diagram below, showing the appropriate probabilities. [3]



b. Find the probability that Maria is late for school. [3]

Consider the following propositions.

p: my Mathematical Studies homework is due tomorrow
q: today is Wednesday

a. Write down in words the compound proposition $\neg p \Rightarrow q$. [2]

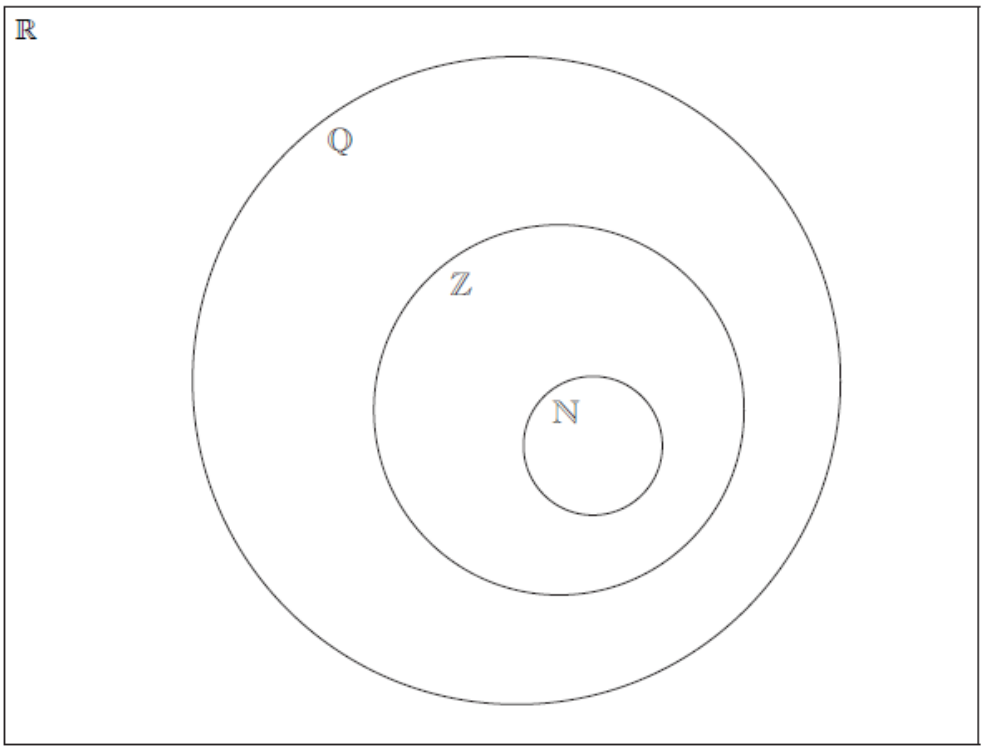
b. Complete the truth table. [3]

p	q	$\neg p$	$\neg p \Rightarrow q$	$\neg p \wedge q$	$(\neg p \Rightarrow q) \vee (\neg p \wedge q)$
T	T	F			
T	F	F			
F	T	T			
F	F	T			

c. State whether the compound proposition $(\neg p \Rightarrow q) \vee (\neg p \wedge q)$ is a tautology, contradiction or neither. [1]

The Venn diagram shows the number sets $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$. Place each of the following numbers in the appropriate region of the Venn diagram.

$\frac{1}{4}$, –3, π, cos 120°, 2.7×10^3, 3.4×10^{-2}



The grades obtained by a group of 20 IB students are listed below:

6	2	5	3	5	5	6	2	6	1
7	6	2	4	2	4	3	4	5	6

a. Complete the following table for the grades obtained by the students. [2]

Grade	Frequency
1	
2	
3	2
4	
5	4
6	
7	1

b. Write down the modal grade obtained by the students. [1]

c. Calculate the median grade obtained by the students. [2]

d. One student is chosen at random from the group. [1]

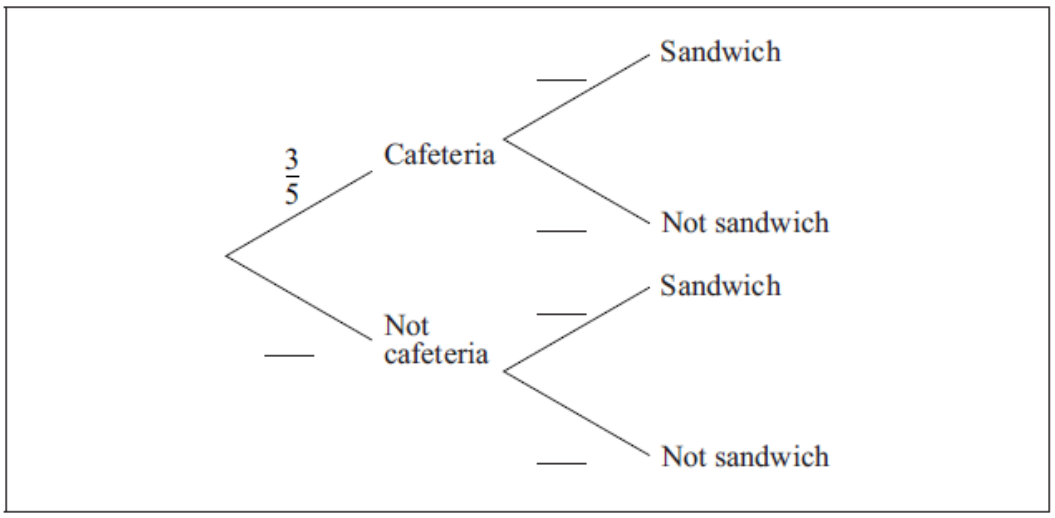
Find the probability that this student obtained either grade 4 or grade 5.

The probability that Tanay eats lunch in the school cafeteria is $\frac{3}{5}$.

If he eats lunch in the school cafeteria, the probability that he has a sandwich is $\frac{3}{10}$.

If he does not eat lunch in the school cafeteria the probability that he has a sandwich is $\frac{9}{10}$.

a. Complete the tree diagram below. [3]



b. Find the probability that Tanay has a sandwich for his lunch. [3]

U is the set of all the **positive** integers less than or equal to 12.

A , B and C are subsets of U .

$$A = \{1, 2, 3, 4, 6, 12\}$$

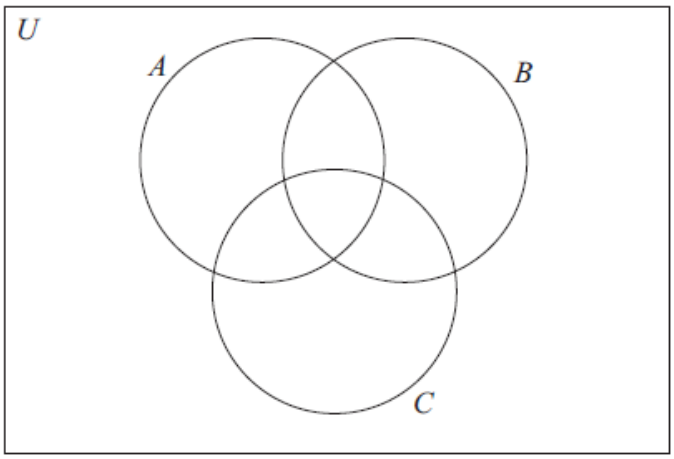
$$B = \{\text{odd integers}\}$$

$$C = \{5, 6, 8\}$$

a. Write down the number of elements in $A \cap C$. [1]

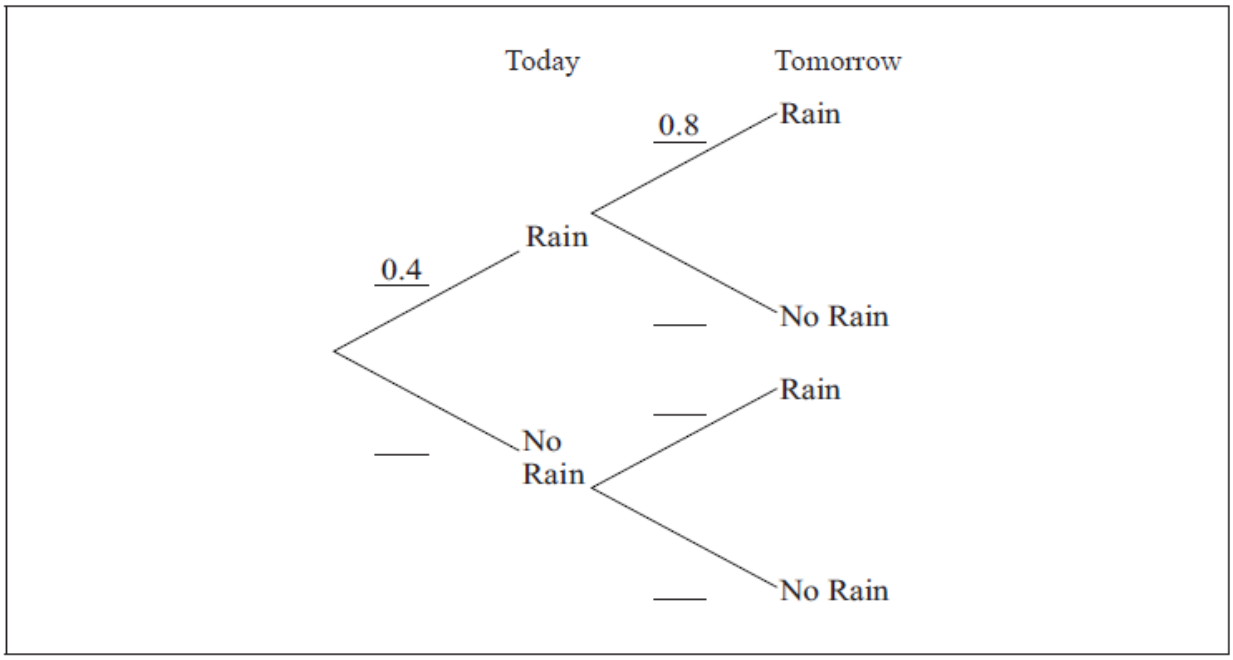
b. List the elements of B . [1]

c. Complete the following Venn diagram with **all** the elements of U . [4]



The probability that it rains today is 0.4 . If it rains today, the probability that it will rain tomorrow is 0.8 . If it does not rain today, the probability that it will rain tomorrow is 0.7 .

a. Complete the tree diagram below. [3]



b. Calculate the probability of rain tomorrow. [3]

A school offers three activities, basketball (B), choir (C) and drama (D). Every student must participate in at least one activity.

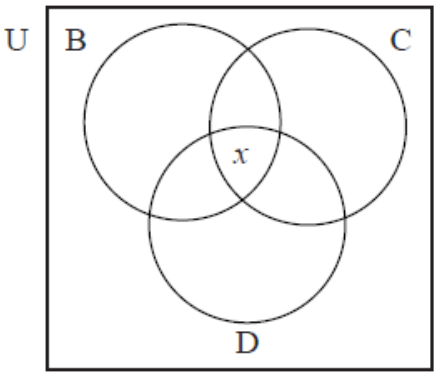
16 students play basketball only.

18 students play basketball and sing in the choir but do not do drama.

34 students play basketball and do drama but do not sing in the choir.

27 students are in the choir and do drama but do not play basketball.

a. Enter the above information on the Venn diagram below. [2]



b. 99 of the students play basketball, 88 sing in the choir and 110 do drama. [1]

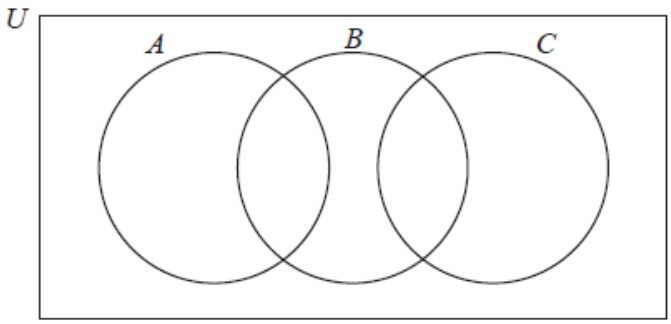
Calculate the number of students x participating in all three activities.

c. 99 of the students play basketball, 88 sing in the choir and 110 do drama. [3]

Calculate the total number of students in the school.

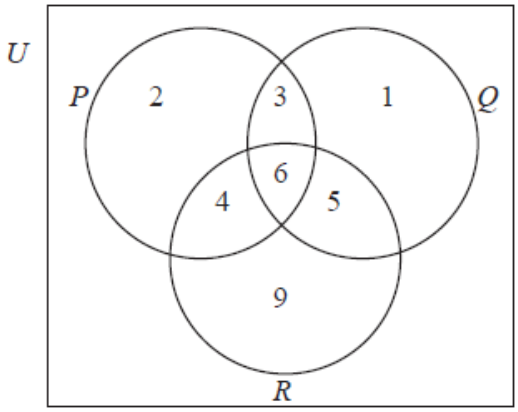
---

a. Shade $(A \cup B) \cap C'$ on the diagram below. [2]



b. In the Venn diagram below, the number of elements in each region is given. [2]

Find $n((P \cap Q) \cup R)$.



c. U is the set of positive integers, $\mathbb{Z}^+$. [2]

E is the set of even numbers.

M is the set of multiples of 3.

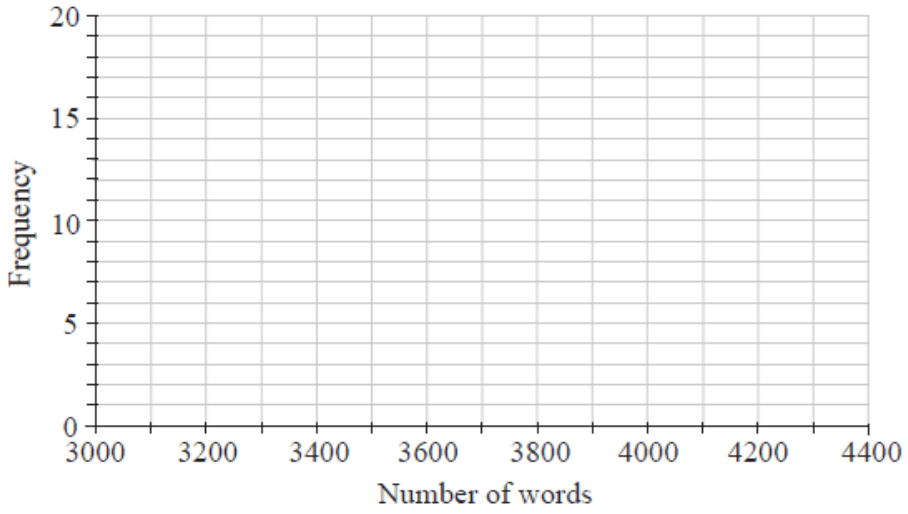
(i) List the first six elements of the set M.

(ii) List the first six elements of the set $E' \cap M$.

The table below shows the number of words in the extended essays of an IB class.

Number of words	$3200 \leq w < 3400$	$3400 \leq w < 3600$	$3600 \leq w < 3800$	$3800 \leq w < 4000$	$4000 \leq w < 4200$
Frequency	2	5	8	17	3

a. Draw a histogram on the grid below for the data in this table. [3]



b. Write down the modal group. [1]

c. The maximum word count is 4000 words. [2]

Write down the probability that a student chosen at random is on or over the word count.

Consider the following Venn diagrams.

Diagram 1

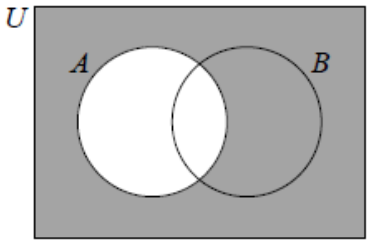


Diagram 2

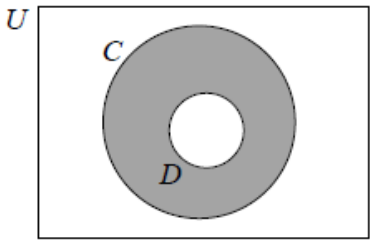
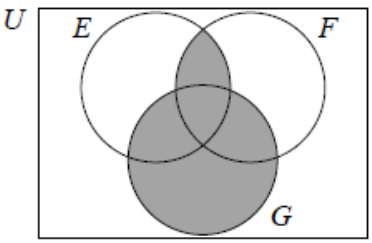


Diagram 3

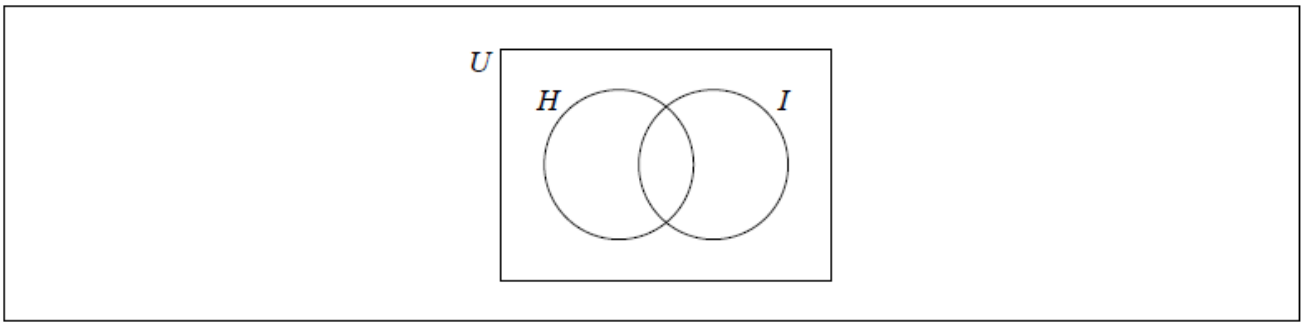


a.i. Write down an expression, in set notation, for the **shaded** region represented by Diagram 1. [1]

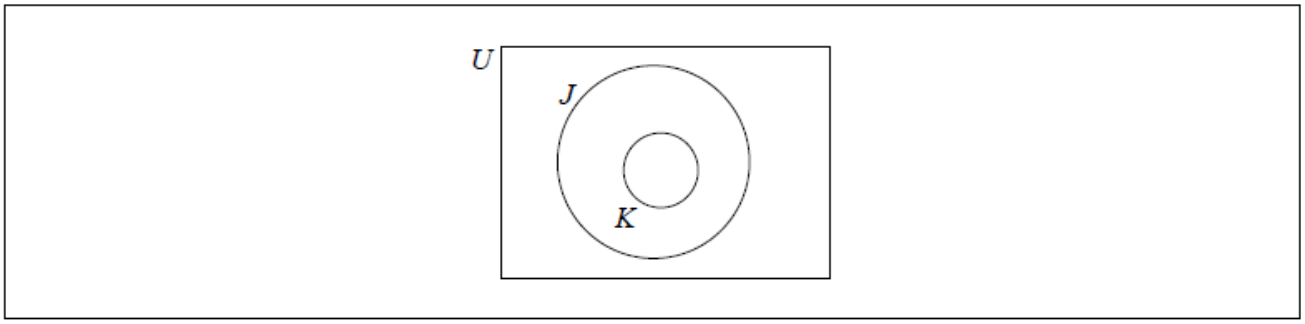
a.ii. Write down an expression, in set notation, for the **shaded** region represented by Diagram 2. [1]

a.iii. Write down an expression, in set notation, for the shaded region represented by Diagram 3. [2]

b.i. Shade, on the Venn diagram, the region represented by the set $(H \cup I)'$. [1]



b.ii. Shade, on the Venn diagram, the region represented by the set $J \cap K$. [1]



All the children in a summer camp play at least one sport, from a choice of football (F) or basketball (B). 15 children play both sports.

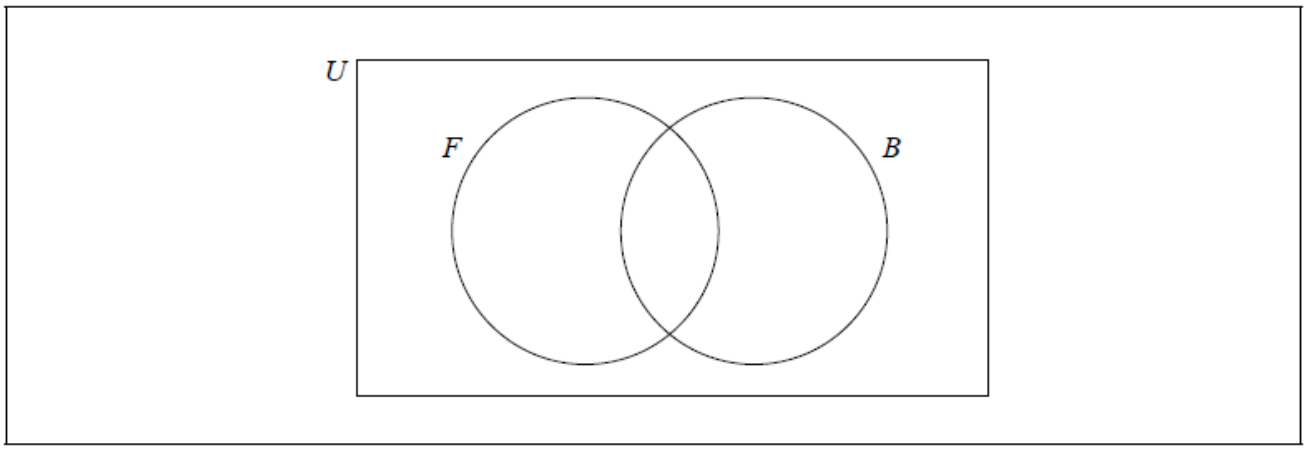
The number of children who play only football is double the number of children who play only basketball.

Let x be the number of children who play only football.

There are 120 children in the summer camp.

a. Write down an expression, in terms of x, for the number of children who play only basketball. [1]

b. Complete the Venn diagram using the above information. [2]



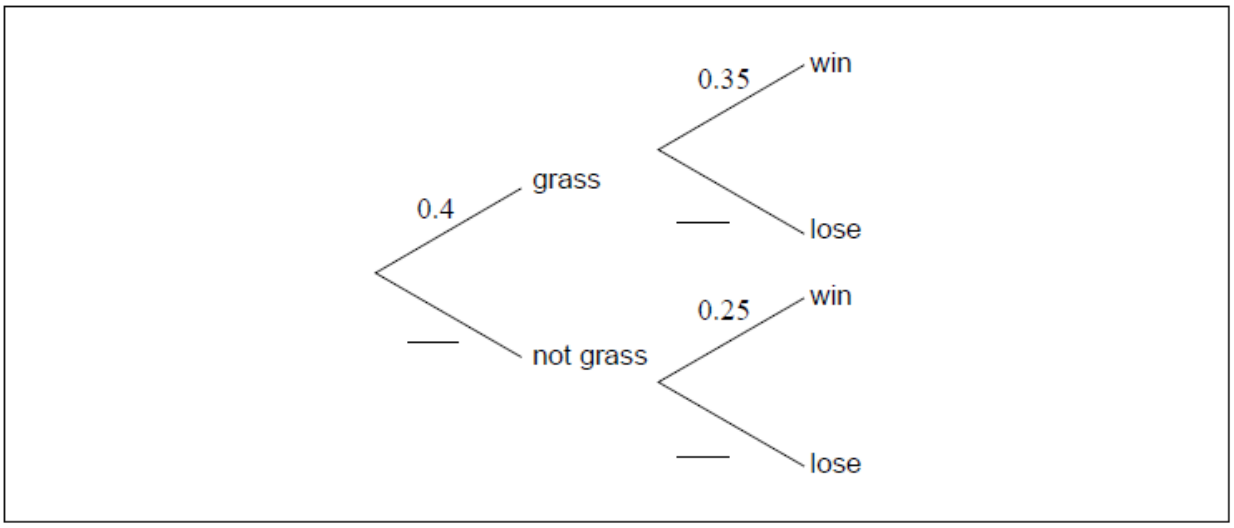
c. Find the number of children who play only football. [2]

d. Write down the value of $n(F)$. [1]

---

a. The probability that Nikita wins a tennis match depends on the surface of the tennis court on which she is playing. The probability that she plays on a grass court is 0.4. The probability that Nikita wins on a grass court is 0.35. The probability that Nikita wins when the court is not grass is 0.25. [3]

Complete the following tree diagram.



b. Find the probability that Nikita wins a match. [3]

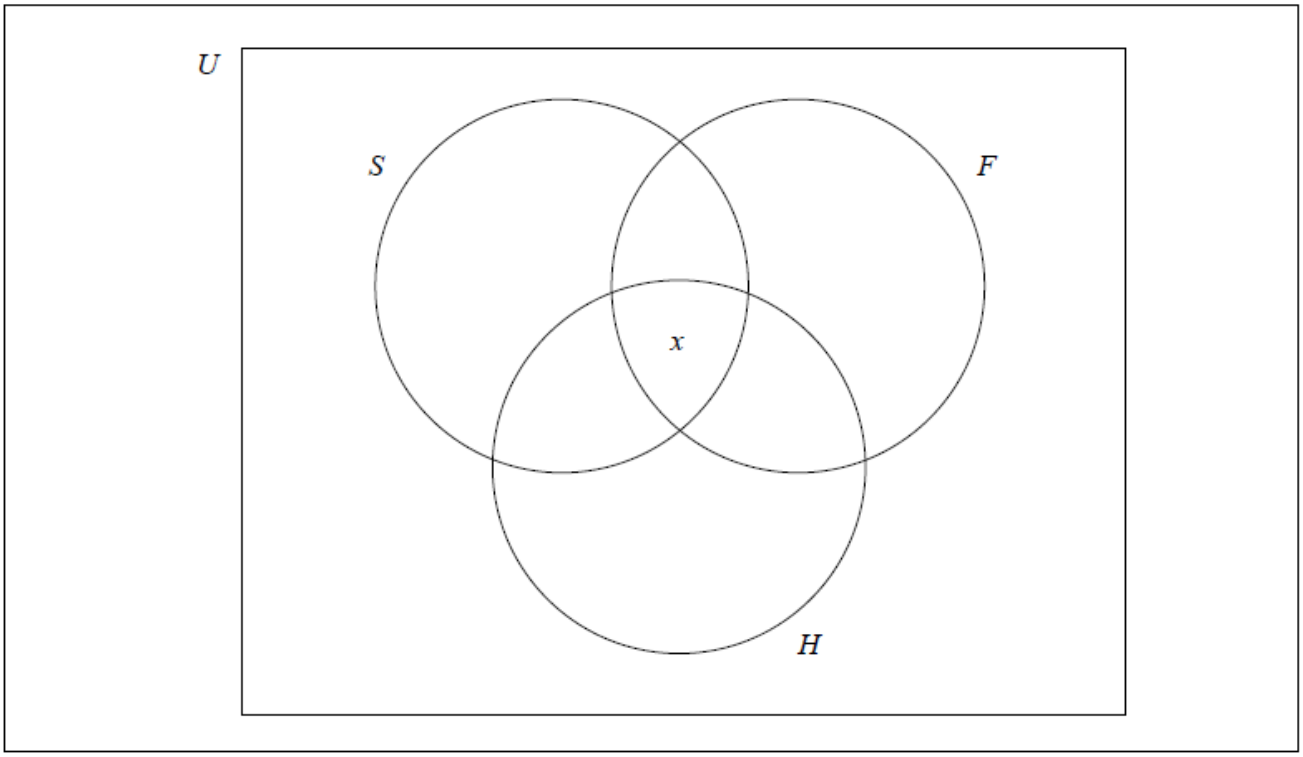
A group of 60 sports enthusiasts visited the PyeongChang 2018 Winter Olympic games to watch a variety of sporting events.

The most popular sports were snowboarding (*S*), figure skating (*F*) and ice hockey (*H*).

For this group of 60 people:

4 did not watch any of the most popular sports,
x watched all three of the most popular sports,
9 watched snowboarding only,
11 watched figure skating only,
15 watched ice hockey only,
7 watched snowboarding and figure skating,
13 watched figure skating and ice hockey,
11 watched snowboarding and ice hockey.

a. Complete the Venn diagram using the given information. [3]



b. Find the value of x. [2]

c. Write down the value of $n((F \cup H) \cap S')$. [1]